Nadar Saraswathi College of Engineering & Technology

NSCET E-LEARNING PRESENTATION

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ELECTRONICS & COMMUNICATION ENGINEERING

II nd YEAR / IVth SEMESTER

EC 8591 – COMMUNICATION THEORY

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UNIT 03

Random Processes

Definition of Random Process

- A deterministic process has only one possible 'reality' of how the process evolves under time.
- In a stochastic or random process there are some uncertainties in its future evolution described by probability distributions.
- Even if the initial condition (or starting point) is known, there are many possibilities the process might go to, but some paths are more probable and others less.
- http://en.wikipedia.org/wiki/Stochastic_process

Many time-varying signals are random in nature:

- Noises
- Image, audio: usually unknown to the distant receiver.
- Random process represents the mathematical model of these random signals.
- Definition: A random process (or stochastic process) is a collection of random variables (functions) indexed by time.

Notation: $X(t, s)$

s: the sample point of the random experiment.

t: time.

Simplified notation:

$X(t)$

The difference between random variable and random process:

- Random variable: an outcome is mapped to a number.
- Random process: an outcome is mapped to a random waveform that is a function of time

We are interested in the ways that these time functions evolve

- correlation
- spectra
- linear systems

•For a fixed sample point sj, X(t, sj) is a realization or sample function of the random process. Simplified Notation:

X (t, s) is denoted as x (t).

• For a fixed time tk, the set of numbers

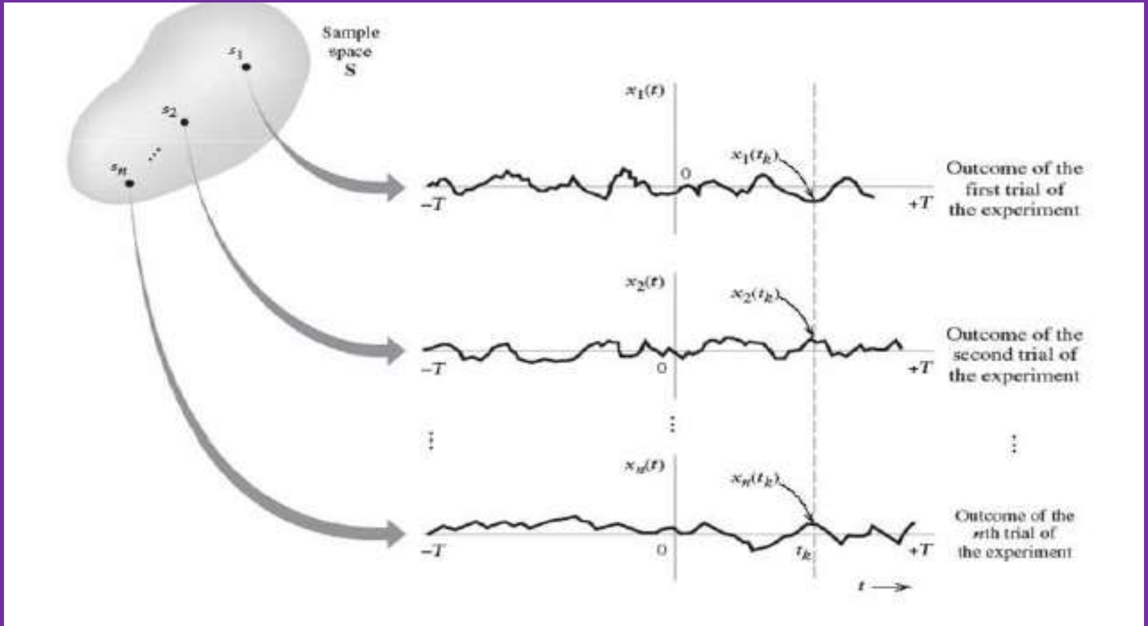
{X(tk, s1), …, X(tk, sn)} = {x1(tk), …, xn(tk)}

is a random variable, denoted by X(tk).

• For a fixed sj and tk, X(tk, sj) is a number.

Pictorial View

Each sample point represents a time-varying function.
Ensemble: The set of all time-varying functions.



Examples of Random Processes

1. $X(t,s) = Y(s)f(t)$ or $X(t,s) = Yf(t)$ for short.

- Y: a random variable.
- f: a deterministic function of parameter t.

2. $X(t,s) = A(s)\cos(2\pi f_0 t + Q)$ or $X(t) = A\cos(2\pi f_0 t + Q)$.

- A: a random variable.
- Q : a random variable.

3. $X(t) = \sum_n X(n) p_n(t - T(n))$

- X(n), T(n): random sequences.
- $p_n(t)$: deterministic waveforms.

Probability Distribution of a Random Process

- For any random process, its probability distribution function is uniquely determined by its finite dimensional distributions.
- The k dimensional cumulative distribution function of a process is defined by

$$F_{X(t_1),\ldots,X(t_k)}(x_1,\ldots,x_k) = P\left(X(t_1) \le x_1,\ldots,X(t_k) \le x_k\right)$$

Stationarity

- In general, the time-dependent N-fold joint pdf’s are needed tocdescribe a random process for all possible N:
- Very difficult to obtain all pdf’s.
- The analysis can be simplified if the statistics are time independent.
- The random process is called first-order stationary if
- $F_{X(t)}(x)$: the CDF of the random process X(t) at a fixed time t.

- The random process is called second-order stationary if the 2nd order CDF is independent of time:

Strict Stationarity

- A strictly stationary process (or strongly stationary process, or stationary process) is a stochastic process whose joint pdf does not change when shifted in time.
- Definition: a random process X(t) is said to be stationary if, for all k, for all /, and for all t1, t2, …, tk,

- An example of strictly stationary process is one in which all X(ti)’s are mutually Independent and Identically Distributed.
- Such a random process is called IID random process.
- Since the joint pdf above does not depend on the times {ti}, the process is strictly stationary.
- An example of IID process is white noise (studied later)
  - Widely used in communications theory

Correlation of Random Processes

$$Cov(X,Y) = E\{[X-\mu_X][Y-\mu_Y]\} = E\{XY\} - \mu_X \mu_Y$$

Consider $X(t_1)$ and $X(t_2)$: samples of X(t) at t_1 and t_2.

$X(t_1)$ and $X(t_2)$ are both random variables

So we can also define their covariance:

$$Cov(X(t_1), X(t_2)) = E\{X(t_1)X(t_2)\} - \mu_{X(t_1)}\mu_{X(t_2)}$$

- Recall: autocorrelation of deterministic energy signals:

$$R_x(\tau) \overset{\Delta}{=} \int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt$$

Similarly, the autocorrelation of deterministic power signal is:

$$R_X(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x^*(t-\tau)dt$$

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Correlation of Random Processes

- For random processes: need to consider probability distributions.

- Note: steps to get $E\{X(t_1)X^*(t_2)\}$:
- 1: For each sample function $X(t, s_j)$, calculate $X(t_1, s_j)\ X^*(t_2, s_j)$.
- 2: Take weighted average over all possible sample functions s_j.
- (See Example 1 later)
- If $X(t)$ is stationary to the 2^{nd} order or higher order, $R_X(t1,t2)$ only depends on the time difference $t_1 - t_2$, so it can be written as a single variable function:

Wide-Sense Stationarity (WSS)

- In many cases we do not require a random process to have all of the properties of the 2nd order stationarity.
- A random process is said to be wide-sense stationary or weakly stationary if and only if

$$R_X(t,s) = E\{X(t)X^*(s)\} = R_X(t-s).$$

Property of Autocorrelation

For real-valued wide-sense stationary X(t), we have:

1. $$R_X(0) = E\{X^2(t)\}.$$

2. $R_X(\tau)$ is even symmetry: $R_X(-\tau) = R_X(\tau)$.

Proof:

3. $R_X(\tau)$ is max at the origin $\tau = 0$.

Thank You