Course/Branch : B.E/Common To All Branches	**Year / Semester** :I/II	Format No.	NAC/TLP-07a.5
Subject Code :MA8251	**Subject Name** :ENGINEERING MATHEMATICS-II	Rev. No.	02
Unit No : 01	**Unit Name** : Matrices	Date	14-11-2017

LECTURE NOTES

1.MATRICES

Simple method to find Characteristics Equations:

Case:1: when A is 2X2 matrix

$\lambda^2 - C_1\lambda + C_2 = 0$

C_1 – Sum of the diagonal elements

C_2 – |A|

Case:2: when A is 3X3 matrix

$\lambda^3 - C_1\lambda^2 + C_2\lambda - C_3 = 0$

C_1 – Sum of the diagonal elements

C_2 – Sum of the minors of diagonal elements

C_3 – |A|

Properties of eigen values:

1. The eigen values of A and A^T are same
2. The Sum of the eigen values of A= Sum of the diagonal elements
3. The product of the eigen values of A = |A|
4. If $K\lambda_1, K\lambda_2, \ldots\ldots K\lambda n$ are the eigen values of KA
5. $\lambda_1^p, \lambda_2^p, \ldots\ldots \lambda_n^p$ are the eigen values of A^p.where P is the positive integer
6. $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots\ldots \frac{1}{\lambda_n}$ are the eigen values of A^{-1} .

Cayley Hamilton theorem

Every Square matrix satisfies its own characteristics equation.

$a_0\lambda^n + a_1\lambda^{n-1} + \cdots \ldots\ldots + a_n = 0$ is the characteristics equation.

of A then

$$a_0\mathrm{A}^n + a_1\mathrm{A}^{n-1} + \cdots \ldots\ldots + a_n I = 0 \ .$$

Method of finding inverse of a matrix

$a_0\lambda^n + a_1\lambda^{n-1} + \cdots \ldots\ldots + a_n = 0$ is the characteristics equation of A then

$$a_0\mathrm{A}^n + a_1\mathrm{A}^{n-1} + \cdots \ldots\ldots + a_n I = 0 \ .$$

Post multiplying equation (I) by A^{-1}

$$a_0\mathrm{A}^{n-1} + a_1\mathrm{A}^{n-2} \ldots\ldots\ldots + a_{n-1}I + a_n\mathrm{A}^{-1} = 0$$

$$\mathrm{A}^{-1} = \frac{-1}{a_n}[\, a_0\mathrm{A}^{n-1} + a_1\mathrm{A}^{n-2} \ldots\ldots\ldots + a_{n-1}I]$$

To reduce the quadratic form to canonical form using orthogonal transformation

Let $Q = X^T AX$ be given Quadradic form

1. Find the matrix "A" of the Quadradic form
2. Find the eigen values of A. Let it be $\lambda_1, \lambda_2, \lambda_3$
3. Find the eigen vectors X_1, X_2, X_3 corresponding to $\lambda_1, \lambda_2, \lambda_3$
4. Check the Vectors X_1, X_2, X_3 for orthogonal
   Find $e_1 = \frac{X_1}{||X_1||}, e_2 = \frac{X_2}{||X_2||}, e_3 = \frac{X_3}{||X_3||}$
5. Find the modal matrix of P whose columns are normalized eigen vectors of A
   (i.e) e_1, e_2, e_3
6. Then X=PY where $Y = (Y_{1,}, Y_{2,}, Y_{3,})^T$ is the required orthogonal transformation
7. Then $Q = X^T AX$.

$=\lambda_1 Y_{1,}{}^2 + \lambda_2 Y_{2,}{}^2 + \lambda_3 Y_{3,}{}^2$ is the required canonical form.

Nature of Quadratic form

1.Rank = No of non-zero eigen values

2.Index = No of positive eigen values

3.Signature = No of positive eigen values - No negative eigen values

4. If all the eigen values of A are positive , then the nature is positive definite

5.If atleast one eigen value of A

is zero and the remaining eigen values are all positive, then the nature is positive semi definite

6. If all the eigen values of A are negative , then the nature is negative definite

7. If atleast one eigen value of A

is zero and the remaining eigen values are all negative, then the nature is negative semi definite

8.If all other cases (i.e) eigen values of A are positive and negative then the nature is indefinite

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LECTURE NOTES

Problem:1

Reduce the Quadratic form $q = 3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2xz.$

Find also rank index signature and nature of the quadratic form.

Solu:

Let $q = 3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2xz.$

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

The characteristic equation is

$c_1 = 9$

$c_2 = 24$

$c_3 = 16$

$\lambda^3 - 9\lambda^2 + 24\lambda - 16 = 0$

Eigen values are 1,4,4.

Case1: $\lambda = 1$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$2x + y + z = 0$$

$$x + 2y - z = 0$$

$$x - y + 2z = 0$$

$$\frac{x}{-3} = \frac{y}{3} = \frac{z}{3}$$

$$X_1 = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

Case2: $\lambda = 4$

$$\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x + y + z = 0$$

$$x - y - z = 0$$

$$x - y - z = 0$$

All the euations are same

Z=0, x = y = 1

$$X_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Case3: $\lambda = 4$

$$\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x + y + z = 0$$

$$x - y - z = 0$$

$$x - y - z = 0$$

$$X_3 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$X_1 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \text{ and } X_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

X_3 *and* X_1 are orthogonal

X_3 *and* X_2 are orthogonal

$$-a + b + c = 0$$

$$a + b + 0c = 0$$

$$\frac{a}{-1} = \frac{b}{1} = \frac{c}{2}$$

$$X_3 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Check:

$$Q = N^T A N$$

$$N = \begin{pmatrix} \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{pmatrix}$$

$$N^T = \begin{pmatrix} \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{pmatrix}$$

$$D = \begin{pmatrix} \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix} \begin{pmatrix} \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$D = X^2 + 4Y^2 + 4Z^2$ is the canonical form

Rank = 3

Index = 3

Signature = 3-0=3

Nature = positive definite.

Problem:2

Using Cayley-Hamilton theorem to find the inverse of

$A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$ and also verify the theorem.

Solu:

The characteristic equation is

$c_1 = 1$

$c_2 = 5$

$c_3 = 1$

$$\lambda^3 - \lambda^2 + 5\lambda - 1 = 0$$

Replace λ by A

$$A^3 - A^2 + 5A - I = 0$$

Multiply by A^{-1}

$$A^2 - A + 5I - A^{-1} = 0$$

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} -1 & -2 & 0 \\ 0 & 1 & -4 \\ 2 & 6 & -9 \end{pmatrix}$$

$$-A^{-1} = -(A^2 - A + 5I)$$

$$A^{-1} = (A^2 - A + 5I)$$

$$A^{-1} = \begin{pmatrix} 3 & -4 & 2 \\ -2 & 1 & 0 \\ -1 & -1 & -1 \end{pmatrix}$$

<u>To verify the theorem</u>:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix},$$

$$A^2 = \begin{pmatrix} -1 & -2 & 0 \\ 0 & 1 & -4 \\ 2 & 6 & -9 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} -5 & -12 & -10 \\ -10 & -23 & 16 \\ -13 & -29 & 17 \end{pmatrix}$$

$$A^3 - A^2 + 5A - I = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Hence verified.